UECM1303 Tutorial 1: Propositional Logic (Semantics and Inference)

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Slide 1: Formal Propositions & Truth Table

1. Let p, q, r and s denote the following statements.

   p: Ali is inside | q: Ali is watching TV
   r: Ali is taking his dinner | s: Ali is riding his bicycle

   (a) Translate the following into English sentences.
      (i) $s \wedge (q \vee \sim r)$
      (ii) $p \rightarrow (q \vee r)$
      (iii) $(p \vee s) \wedge (p \rightarrow q)$
      (iv) $\sim s \rightarrow (p \wedge (q \vee r))$

   (b) Translate the following into logical notation.
      (i) Ali is neither inside nor is he riding his bicycle.
      (ii) Ali is inside, and he is taking his dinner while watching TV.
      (iii) Ali is not watching TV only if he is outside.
      (iv) Ali is inside and taking his dinner implies that he is not riding his bicycle.
      (v) If Ali is not watching TV, then if he is not taking his dinner, he is outside.

2. Given that p and q are true and r, s and t are false, find the truth value of each statement below.

   (a) $(p \vee \sim q) \rightarrow (r \wedge s \wedge t)$
   (b) $(q \rightarrow (r \rightarrow s)) \wedge ((p \rightarrow s) \rightarrow (\sim t))$

3. If statement q is true, determine all truth values assignments for the statements p, r and s for which the truth value of the following statement is true:

$$(q \rightarrow [(\sim p \vee r) \wedge \sim s]) \wedge [\sim s \rightarrow (\sim r \wedge q)].$$

4. Give the negation, converse, inverse and contrapositive of each the following statements.

   (a) I will pass the course if I work hard.
   (b) If $A = B \cap C$, then $A \subset C$.
   (c) If $-2 < 4$ and $3 + 8 = 11$, then $\sin(\pi/2) = 1$.

5. Construct truth tables for the following statement forms:

   (a) $(p \leftrightarrow q) \leftrightarrow (\sim p \leftrightarrow \sim q)$
   (b) $\sim p \rightarrow (p \vee q)$
   (c) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

Then determine whether each of the above statement forms is a tautology, a contingency or a contradiction.

6. Determine whether the following statement forms are tautologies.

   (a) $p \to [q \to (p \wedge q)]$

   (b) $(p \vee q) \to (q \to q)$

   (c) $(p \vee q) \to [q \to (p \wedge q)]$

Slide 1: Logical Equivalence & Logical Implication

7. Answer true or false.

   (a) An equivalent way to express the converse of "p is sufficient for q" is "p is necessary for q". . . . . $\square$

   (b) An equivalent way to express the inverse of "p is necessary for q" is "$\sim q$ is sufficient for $\sim p$". . . . . $\square$

   (c) An equivalent way to express the contrapositve of "p is necessary for q" is "$\sim q$ is necessary for $\sim p$". . . . . $\square$

8. Rewrite the following statements in if-then form.

   (a) Fix my ceiling or I won't pay my rent.

   (b) Study hard or I won't pass Discrete Mathematics.

   (c) Catching the 7am bus is a sufficient condition for my being on time for school.

   (d) Doing homework regularly is a necessary condition to pass the course.

   (e) Ali studies calculus only if he is a math major.

   (f) P is a square only if P is a rectangle.

   (g) n is divisible by 6 is a sufficient condition for n to be divisible by 2 and n is divisible by 3.

9. Determine whether the 2 statements forms are equivalent.

   (a) $p \to (q \to r)$ and $(p \to q) \to r$.

   (b) $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$.

   (c) $(p \vee q) \to r$ and $(p \to r) \wedge (q \to r)$.

10. Explain why the statement "If today is not cold, then today is cold" is logically equivalent to the statement "Today is cold".

11. (a) Show that the following statement forms are all logically equivalent.

$$p \to (q \vee r), \quad (p \wedge \sim q) \to r \quad \text{and} \quad (p \wedge \sim r) \to q.$$

   (b) Use the logical equivalences in (a) to rewrite the sentence "If n is prime, then n is odd or n is 2." in 2 different ways.

12. Use the laws of logical equivalence to show the following:

   (a) $(p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) \equiv p$.

   (b) $\sim (p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$.

   (c) $\sim ((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$.

   (d) $(\sim p \vee \sim q) \to (p \wedge q \wedge r) \equiv p \wedge q$.

13. Simplify the following statement to a statement with no more than 3 logical connectives involving $\sim$, $\vee$ and $\wedge$ by stating the law used in each step of the simplification:

$$[[[(p \wedge q) \wedge r] \vee [(p \wedge q) \wedge \sim r]] \vee \sim q] \rightarrow s.$$

14. Verify that

    (a) $(p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \equiv \sim (p \vee q)$

    (b) $p \vee q \vee (\sim p \wedge \sim q \wedge r) \equiv p \vee q \vee r$

    (c) $(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p) \equiv (p \rightarrow q) \wedge (q \rightarrow r) \wedge (r \rightarrow p)$

15. In logic circuit design, one of the basic logic gate is the NAND gate. It is logically equivalent to $\sim (p \wedge q)$ and denoted by $(p \uparrow q)$ for any statements p and q.

    (a) Represent the logic gates (i) $\sim p$ and (ii) $p \rightarrow q$ using the NAND gate.

    (b) Are $p \uparrow (q \uparrow r)$ and $(p \uparrow q) \uparrow r$ logically equivalent?

16. Use comparison tables to determine whether the argument forms are valid.

    (a)
    $$\begin{array}{ll} & p \rightarrow (q \vee r) \\ & \sim q \vee \sim r \\ \hline \therefore & \sim p \vee \sim r \end{array}$$

    (b) $(p \wedge q) \rightarrow \sim r,\ p \vee \sim q,\ \sim q \rightarrow p / \therefore \sim r$

17. Write the symbolic form of each of the following arguments and then determine its validity.

    (a) If Tom is not on team A, then Hua is on team B.
    If Hua is not on team B, then Tom is on team A.
    Therefore Tom is not on Team A or Hua is not on Team B.

    (b) If I graduate this semester, then I will have passed Calculus.
    If I do not study Calculus for 5 hours a week, then I will not pass Calculus.
    If I study Calculus for 5 hours a week, then I cannot play basketball.
    Therefore, if I play basketball, I will not graduate this semester.

    (c) If f is integrable, then g or h is differentiable.
    If g is not differentiable, then f is not integrable but it is bounded.
    If f is bounded, then g or h is differentiable.
    Therefore, g is differentiable.

Slide 1: Rules of Inference

18. Use the rules of inference to show that $p \wedge \sim s,\ q \rightarrow (r \rightarrow s) \vdash (p \rightarrow q) \rightarrow \sim r$.

19. Show that $p \rightarrow q \vdash p \rightarrow (r \rightarrow (s \rightarrow q))$.